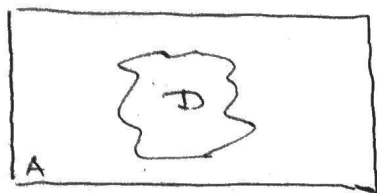


INTEGRACIJA NAD PROIZVOLJNOM OBLAŠĆU



$$f: D \rightarrow \mathbb{R}, D \subseteq \mathbb{R}^2$$

$$\bar{f}: A \rightarrow \mathbb{R}, A \subseteq \mathbb{R}^2 : \quad \bar{f}(x) = \begin{cases} f(x), & x \in D \\ 0, & x \notin D \end{cases}$$

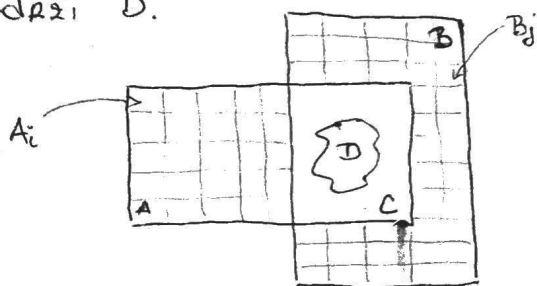
Def: Dvostruki integral funkcije f na ograničenom skupu $D \subseteq \mathbb{R}^2$ definišemo sa:

$$\iint_D f(x,y) dx dy = \iint_A \bar{f}(x,y) dx dy$$

gde je A proizvoljni pravougaonik koji sadrži skup D , a funkcija \bar{f} neprekidno proširenje nule funkcije f .

Napomena: Pošto je D ograničen, uvek će postojati pravougaonik koji ga sadrži. (sledi iz def. ograničenog skupa)

Napomena: Ova definicija ne zavisi od izbora pravougaonika koji sadrži D .



$$C = A \cap B$$

$$A = C \cup A_1 \cup \dots \cup A_p$$

$$B = C \cup B_1 \cup \dots \cup B_q$$

$$\bar{f}(x,y) = \begin{cases} f(x,y), & (x,y) \in D \\ 0, & (x,y) \notin D \end{cases}$$

\Rightarrow Na $A_1, \dots, A_p, B_1, \dots, B_q$ funkcija \bar{f} ima vrednost 0
 \Rightarrow integrabilna

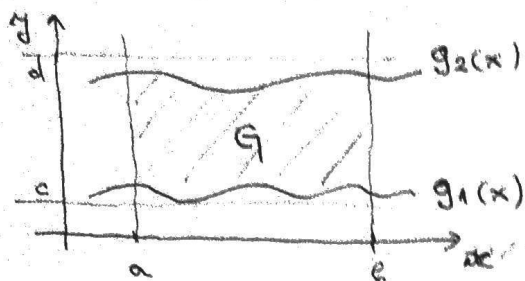
$$\iint_A \bar{f}(x) dx = \iint_C \bar{f}(x) dx + \underbrace{\sum_{i=1}^p \iint_{A_i} \bar{f}(x) dx}_0$$

$$\iint_B \bar{f}(x) dx = \iint_C \bar{f}(x) dx + \underbrace{\sum_{j=1}^q \iint_{B_j} \bar{f}(x) dx}_0$$

$$\Rightarrow \iint_A \bar{f}(x) dx = \iint_B \bar{f}(x) dx = \iint_C \bar{f}(x) dx$$

$$f: G \rightarrow \mathbb{R}, \quad G \subseteq \mathbb{R}^2$$

$$G = \{(x, y) \in \mathbb{R}^2 \mid a \leq x \leq b, \quad g_1(x) \leq y \leq g_2(x)\}$$



$$\bar{f}(x, y) = \begin{cases} f(x, y), & a \leq x \leq b, \quad g_1(x) \leq y \leq g_2(x) \\ 0, & \text{inače} \end{cases}$$

$$\iint_G f(x, y) dx dy \stackrel{\text{def}}{=} \iint_{[a, b] \times [c, d]} \bar{f}(x, y) dx dy$$

$$= \int_a^b \left(\int_c^d \bar{f}(x, y) dy \right) dx$$

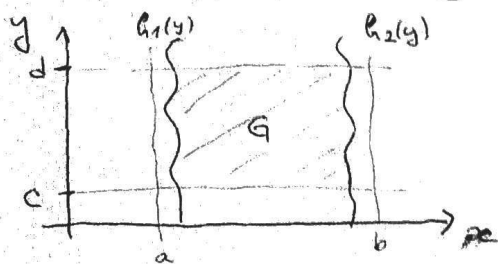
$$= \int_a^b \left(\underbrace{\int_c^{g_1(x)} \bar{f}(x, y) dy}_0 + \int_{g_1(x)}^{g_2(x)} \bar{f}(x, y) dy + \underbrace{\int_{g_2(x)}^d \bar{f}(x, y) dy}_0 \right) dx$$

$$= \int_a^b \left(\int_{g_1(x)}^{g_2(x)} \bar{f}(x, y) dy \right) dx$$

$$= \int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x, y) dy \right) dx$$

$$f: G \rightarrow \mathbb{R}, \quad G \subseteq \mathbb{R}^2$$

$$G = \{(x, y) \in \mathbb{R}^2 \mid h_1(y) \leq x \leq h_2(y), \quad c \leq y \leq d\}$$



Analogni postupkom: $\iint_G f(x, y) dx dy = \int_c^d \left(\int_{h_1(y)}^{h_2(y)} f(x, y) dx \right) dy$

Napomena: Spoljašnji integral mora imati konstantne granice!